ASSIGNMENT 1

GROUP 3

SECTION 03 - 2024/2025

SECI1013 (DISCRETE STRUCTURE)

LECTURER : DR. MUHAMMADALIIFBINAHMAD

DATE OF SUBMISSION : 19th NOVEMBER 2024

	NAME:	MATRICS NO.
1.	IDA YATULLAILIYEH BINTI AMRUN	A24CS0084
2.	PARTHIV GUNALAN	A24CS0178
3.	AHMAD WILDAN BIN MAZANI	A24CS0222
4.	AHMED ABDELHADI MOHAMED ZIEN	A22EC4002

Question 1

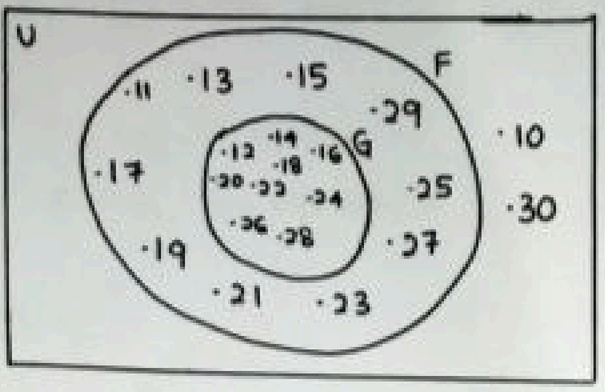
1a. $F = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$|F| = 19$

1b. $G = \{12, 14, 16, 18, 20, 22, 24, 26, 28\}$

$|G| = 9$

1c.



1d. $F \oplus G = (F - G) \cup (G - F)$

$F \oplus G = (F \cap G') \cup (G \cap F')$

$G \cap F' = \{\emptyset\}$

$F \cap G' = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$(F \cap G') \cup (G \cap F') = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$|F \oplus G| = 10$

Question 2

2a. $|A| = 3$

$|P(A)| = 2^3$

$|P(A)| = 8$

2b. $A \cap B \cup C = \{e, n, s, t\}$

2c. $A - B = A \cap B'$

$A - B = \{b, u\}$

2d.

$B \times C = \{(e,e), (e,n), (e,t), (s,e), (s,n), (s,t), (t,e), (t,n), (t,t)\}$

Question 3

(a) It is not a proposition.

(b) It is a proposition and it is true.

(c) It is a proposition and it is true.

(d) It is a proposition and it is false.

(e) It is a proposition and it is true.

Question 4

4a. $(p \to q) \wedge (\neg p \leftrightarrow \neg q)$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \leftrightarrow \neg q$	$(p \to q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

4b. $(p \leftrightarrow q) \vee (\neg p \to \neg q)$

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \leftrightarrow q) \vee (\neg p \to \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

CS CamScanner

Question 5

$A = \neg p \wedge (\neg q \vee \neg r)$

$B = p \vee (q \wedge r)$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \vee \neg r$	$q \wedge r$	A	B
T	T	T	F	F	F	F	T	F	T
T	T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F
F	F	F	T	T	T	T	F	T	F

Hence, $A \not\equiv B$

Question 6

$A = p \wedge (p \vee q)$

$B = p \vee (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	F	F	F	F

$p \wedge (p \vee q) \equiv p \vee (p \wedge q)$

Thus, $A \equiv B$

7. Let $P(x)$, $Q(x)$ and $R(x)$ be the statements

"x is a student"

"x is smart"

"x is shy"

a. Some students are shy:

$$\exists x\,(P(x) \wedge R(x))$$

b. All smart people are not shy:

$$\forall x\,(Q(x) \rightarrow \neg R(x))$$

Question 8

Let $P(x)$ = x is a negative number

Let $Q(x)$ = x^2 is a positive number

Thus, $\forall x\,(P(x) \longrightarrow Q(x))$

Let n is a positive integer.

$x = -n$,

$x^2 = (-n)^2$

$x^2 = (-n)(-n)$

$x^2 = n^2$

this shows that when x is a negative number,
x^2 will be a positive number.

therefore, if x is a negative number, x^2 will be positive number.

example :

$x = -2$

$x^2 = (-2)^2$

$x^2 = (-2)(-2)$

$x^2 = 4$

Question 9

$C \cap (D \cap C') = \{\}$

$\Leftarrow$

- Contradiction : Assume $C \cap (D \cap C') \neq \{\}$
- Means there is element x exist such $x \in C \cap (D \cap C')$
- $x \in C$ and $x \in (D \cap C')$

  if $x \in (D \cap C')$, then $x \in D$ and $x \in C'$.

  - $x \in C$
  - $x \in D$
  - $x \in C'$

- $x \in C'$ means $x \notin C$
- x cannot be in both C and C'
- so the contradiction is false.

Hence the statement $C \cap (D \cap C')$ is True.

10. $a R b \rightleftharpoons |a-b| = 2$

matrix example : transpose matrix

$$M_R = \begin{array}{c|c} a \backslash b & \begin{array}{ccccc} -2 & -1 & 0 & 1 & 2 \end{array} \\ \begin{array}{c} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad M_{T_R} = \begin{array}{c|c} & \begin{array}{ccccc} -2 & -1 & 0 & 1 & 2 \end{array} \\ \begin{array}{c} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Relation R is :

1. Not reflexive because a and b are not same value $a \neq b$
   as example : $|2-2| \neq 2$. Thus, the relation is not reflexive.

2. Irreflexive because a and b are not same value $a \neq b$,
   Hence, the relation is IRreflexive

3. Symmetric because for every $(a,b) \in R$, $(b,a) \in R$, and $M_R = M_{T_R}$
   Example , $|2-0| = 2$ and $|0-2| = 2$, thus the relation is symmetric

4. Not asymmetric because for every $(a,b) \in R$, there is also $(b,a) \in R$, $M_R = M_{T_R}$
   Hence, the relation is not asymmetric

5. Not antisymmetric because for every $(a,b) \in R$, there is also
   $(b,a) \in R$. Hence, relation R is not antisymmetric

6. Not Transitive because the product of boolean is not the same
   as the matrix of relation, $M_R \odot M_R \neq M_R$. Hence, the relation is not transitive

∴ relation R is
reflexive : NO
Irreflexive : YES
Symmetric : YES
Asymmetric : NO
Antisymmetric : NO
Transitive : NO

Question 11

$A = \{a, b, c, d\}$

$R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)\}$

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

= all the main diagonal of matrix, M_R is 1. Thus, matrix of relation, M_R is reflexive

$$M_R^T = \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

= the tranpose matrix, M_R^T is not the same as the matrix of relation, M_R. Hence, the matrix of relation, M_R is not symmetric

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

= The product of boolean is not the same as matrix of relation, M_R. Therefore, the matrix of relation, M_R is not transitive.

Hence, the relation R is not an equivalence relation.

Question 12

12a. $f(x,y) = (2x-y, x-2y)$; $(x,y) \in R \times R$

$(x_1, y_1) = (x_2, y_2)$

$(2x_1 - y_1, x_1 - 2y_1) = (2x_2 - y_2, x_2 - 2y_2)$

$2x_1 - y_1 = 2x_2 - y_2 \longrightarrow$ equation 1

$x_2 - 2y_2 = x_1 - 2y_1 \longrightarrow$ equation 2

from equation 1

$2x_1 - y_1 = 2x_2 - y_2$

$2x_1 - 2x_2 = y_1 - y_2$

$2(x_1 - x_2) = y_1 - y_2 \longrightarrow$ equation 3

from equation 2

$x_1 - 2y_1 = x_2 - 2y_2$

$x_1 - x_2 = 2y_1 - 2y_2$

$x_1 - x_2 = 2(y_1 - y_2)$

Substitute equation 3 into equation 2

$x_1 - x_2 = 2(2(x_1 - x_2))$

$x_1 - x_2 = 4(x_1 - x_2)$

$x_1 - x_2 = 4x_1 - 4x_2$

$-3x_1 = -3x_2$

$x_1 = x_2$

Substitute $x_1 = x_2$ into equation 3

$2(x_2 - x_2) = y_1 - y_2$

$2(0) = y_1 - y_2$

$0 = y_1 - y_2$

$y_2 = y_1$

Hence,

$(x_1, y_1) = (x_2, y_2)$

Therefore,

f is one to one function.

12b. Find f^{-1}

Assume that $f(x,y) = (2x-y, x-2y)$ will give answer (a,b)

Thus,

$2x-y = a$ and $x-2y = b$

$2x-y=a \longrightarrow$ equation 1

$x-2y=b \longrightarrow$ equation 2

<u>From equation 1</u>

$2x-y=a$

$y=2x-a \longrightarrow$ equation 3

<u>Substitute equation 3 into equation 2</u>

$x-2(2x-a)=b$

$x-4x+2a=b$

$-3x+2a=b$

$-3x=b-2a$

$3x=2a-b$

$x=\dfrac{2a-b}{3}$

<u>Substitute $x=\frac{2a-b}{3}$ into equation 3</u>

$y=2\left(\dfrac{2a-b}{3}\right)-a$

$y=\dfrac{4a-2b}{3}-a$

$y=\dfrac{4a-2b}{3}-\dfrac{3a}{3}$

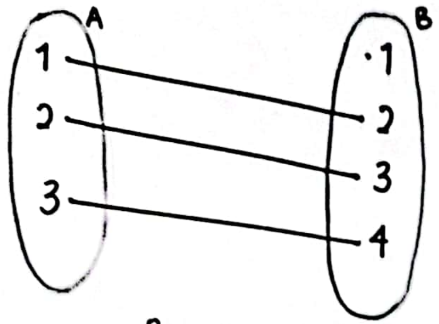
$y=\dfrac{a-2b}{3}$

Hence,

$$f^{-1}(a,b)=\left(\frac{2a-b}{3}, \frac{a-2b}{3}\right)$$

Question 13

13a.



$f: A \to B$

13b.

three ordered pairs thot define
$g: A \to C$, that is onto

$= (1,2), (2,2), (2,1)$

14. $g(x) = x-1$

$f(x) = x^3$

i) ~~$gf(x) =$~~

$g \circ f = gf(x)$

$g \circ f = (x^3) - 1$

$g \circ f = x^3 - 1$ #

$f \circ g = f(g(x))$

$= (x-1)^3$

$= (x-1)^2 (x-1)$

$= (x^2 - 2x + 1)(x-1)$

$= x^3 - 2x^2 + x - x^2 + 2x - 1$

$= x^3 - 3x^2 + 3x - 1$

ii) sub $x = 2$ into both function;

$gf(x) = x^3 - 1$

$= 2^3 - 1$

$= 7$

$fg(x) = x^3 - 3x^2 + 3x - 1$

$= 2^3 - 3(2)^2 + 3(2) - 1$

$= 1$

$\therefore gf(x) \neq fg(x)$ #

Question 15

Let a_n be the number of strings that do not contain 01

when $n = 1, a_n = 2 \rightarrow \{0\}, \{1\}$

$n = 2, a_n = 4 \rightarrow \{00\}, \cancel{\{01\}}, \{10\}, \{11\}$

$= 4 - 1 \rightarrow \{00\}, \{10\}, \{11\}$

$= 3 \rightarrow$

For a_n to not include 01;

Problem 1: when strings end with 0, add 0 to the end.

Problem 2: when strings end with 1, add 1 to the end.

Example P1: 100, 000, 1100, 1001

Example P2: 111, 011, 0011, 0111

Two problems;

$a_n = n + 1$

$a_{n-1} = (n-1) + 1$

$= n$

$n = a_{n-1}$ —②

$n = a_n - 1$. —①

$a_n - 1 = a_{n-1}$

$a_n = a_{n-1} + 1, \quad n \geq 2, \quad a_1 = 2$

Question 16

- $f_1 = 0$
- $f_2 = 1$
- $f_3 = 1$
- $f_n = f_{n-2} + f_{n-3}$ for $n \geq 4$

  0, 1, 1, 1, 2

- Input : n
- Output = f(n)
- f(n) {

```
    if (n = 1)
      return 0
    else if (n = 2 or n = 3)
      return 1
    return f(n-2) + f(n-3)
}
```